Stuff you MUST know cold for AP Calculus!

Note: Letters like a, b, c, d, m, and n are traditionally used to represent constants.
Letters like f, g, h, u, v, x, and y and traditionally used to represent variables or functions.

Basic Derivatives

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$
$$\frac{d}{dx}(\sin x)=\cos x$$
$$\frac{d}{dx}(\cos x)=-\sin x$$
$$\frac{d}{dx}(\tan x)=\sec^2 x$$
$$\frac{d}{dx}(\cot x)=-\csc^2 x$$
$$\frac{d}{dx}(\sec x)=\sec x\tan x$$
$$\frac{d}{dx}(\csc x)=-\csc x\cot x$$
$$\frac{d}{dx}(\ln x)=\frac{1}{x}$$
$$\frac{d}{dx}\left(e^x\right)=e^x$$

More Derivatives

$$\frac{d}{dx}\left(\sin^{-1} x\right)=\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\cos^{-1} x\right)=\frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\left(\tan^{-1} x\right)=\frac{1}{1+x^2}$$
$$\frac{d}{dx}\left(\cot^{-1} x\right)=\frac{-1}{1+x^2}$$
$$\frac{d}{dx}\left(\sec^{-1} x\right)=\frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(\csc^{-1} x\right)=\frac{-1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}\left(a^x\right)=a^x\ln a$$
$$\frac{d}{dx}\left(\log_a x\right)=\frac{1}{x\ln a}$$

If a function is differentiable, **then** it is continuous.

Definitions of Derivative

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

Differentiation Rules

Chain Rule

If $f(x)=g(h(x))$, then
$f'(x)=g'(h(x))\cdot h'(x)$.

OR

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

Product Rule

$$\frac{d}{dx}(uv)=v\frac{du}{dx}+u\frac{dv}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$$

"PLUS A CONSTANT!"

The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx=F(b)-F(a)$$
where $F'(x)=f(x)$

Corollary to the FTC

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt=f(g(x))\cdot g'(x)$$

Intermediate Value Theorem

If function f is continuous for all x in the closed interval $[a,b]$, and y is a number between $f(a)$ and $f(b)$, then there is a number $x=c$ in (a,b) for which $f(c)=y$.

Rolle's Theorem

If f is differentiable for all values of x in the open interval (a,b), and f is continuous at $x=a$ and at $x=b$, and $f(a)=f(b)=0$, then there is at least one number $x=c$ in (a,b) such that $f'(c)=0$.

Mean Value Theorem

If f is differentiable for all values of x in the open interval (a,b), and f is continuous at $x=a$ and $x=b$, then there is at least one number $x=c$ in (a,b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$

Trapezoidal Rule

$$\int_a^b f(x)dx=\frac{1}{2}\left(\frac{b-a}{n}\right)\left[f(x_0)+2f(x_1)+\cdots+2f(x_{n-1})+f(x_n)\right]$$

Distance, Velocity, and Acceleration

If distance, velocity and acceleration are represented by s, v, and a, respectively, then $a=\frac{dv}{dt}=\frac{d^2s}{dt^2}$ and $v=\frac{ds}{dt}$.

$$\text{change in } v=\int_{t_0}^{t_1} a\,dt$$
$$\text{change in } s=\int_{t_0}^{t_1} v\,dt$$

l'Hôpital's Rule

If $\lim_{x\to a}\frac{f(x)}{g(x)}\to\frac{0}{0}$ or $\frac{\infty}{\infty}$

then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$