

Find two positive numbers that satisfy the given requirements.

1. The sum is 243 and the product is a maximum.
2. The product is 198 and the sum is a minimum.
3. The product is 198 and the sum of the first plus four times the second is a minimum.
4. The sum of the first and twice the second is 100 and the product is a maximum.

Find the length and width of a rectangle that has the given perimeter and has a maximum area.

5. Perimeter: 125 feet
6. Perimeter: 200 feet

Find the length and width of a rectangle that has the given area and has a minimum perimeter.

7. Area: 80 square feet
8. Area: 100 square feet

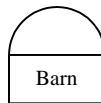
Find the point on the graph of the function closest to the given point.

9. $f(x) = \sqrt{x}$, (3, 1)
10. $g(x) = x^3$, (0, 5)

11. A rancher has 500 feet of fencing with which to enclose four adjacent corrals (see diagram below). What dimensions should be used so that the enclosed area will be a maximum?



12. A rancher has 100 feet of fencing with which to enclose a semi-circular area adjacent to a barn (see diagram below). What radius should be used so that the enclosed area will be a maximum?



13. An open box is to be made from an 8.5 in by 11 in piece of paper by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box of maximum volume.

14. A Norman window is constructed by adjoining a semi-circle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

15. A rancher has 400 feet of fencing with which to enclose two rectangular corrals adjacent to one triangular corral (see diagram below). What dimensions should be used so that the enclosed area will be a maximum?

