AP Calculus
Section 7.3: Other Differential Equations for Real-World Applications

p.321 #1

a)

$$\frac{dM}{dt} = 100 - S$$

b)

$$S = k \cdot M$$

$$\frac{dM}{dt} = 100 - kM$$

c)

$$\frac{dM}{100 - kM} = dt$$

$$\int \frac{dM}{100 - kM} = \int dt$$

$$\frac{-1}{k}\ln\left|100 - kM\right| = t + C$$

$$\ln\left|100 - kM\right| = -kt - kC$$

$$\left|100 - kM\right| = e^{-kt-kC}$$

$$100 - kM = e^{-kt} \cdot e^{-kC}$$

$$100 - kM = C_1 e^{-kt}$$

$$-kM = -100 + C_1 e^{-kt}$$

$$M = \frac{1}{k}\left(100 - C_1 e^{-kt}\right)$$

$$0 = \frac{1}{k}\left(100 - C_1 e^{0}\right)$$

$$C_1 = 100$$

$$M = \frac{1}{k}\left(100 - 100e^{-kt}\right)$$

$$M = \frac{100}{k}\left(1 - e^{-kt}\right)$$

d)

$k = 0.02$

$$M = 5000\left(1 - e^{-0.02t}\right)$$

e) (graph)

f)

days	in cabinet?	payed in?	spent?
30	$2255.94	$3000.00	$744.06
60	$3494.03	$6000.00	$2505.97
90	$4173.51	$9000.00	$4826.49

g)

365.2522 days in a year

$$M\left(365.2522\right) = \$4996.64$$

$$\left.\frac{dM}{dt}\right|_{t=365.2522} = 100 - 0.02\left(4996.6393\right)$$

$$\frac{dM}{dt} = 0.067$$

increasing about 7 cents per day

h)

$$\lim_{t\to\infty} 5000\left(1 - e^{-0.02t}\right)$$

$$= \lim_{t\to\infty} 5000\left(1 - \frac{1}{e^{0.02t}}\right) = 5000$$

p.321 #3

a)

$$E = RI + L\left(\frac{dI}{dt}\right)$$

b)

$$L\frac{dI}{dt} = E - RI$$

$$\frac{L}{E - RI}\,dI = dt$$

$$\int \frac{L}{E - RI}\,dI = \int dt$$

$$-\frac{L}{R}\ln\left|E - RI\right| = t + C$$

$$\ln\left|E - RI\right| = -\frac{R}{L}t - \frac{R}{L}C$$

$$E - RI = e^{-\frac{R}{L}t - \frac{R}{L}C}$$

$$E - RI = e^{-\frac{R}{L}t} \cdot e^{-\frac{R}{L}C}$$

$$E - RI = C_1 e^{-\frac{R}{L}t}$$

$$-RI = C_1 e^{-\frac{R}{L}t} - E$$

$$I = \frac{1}{R}\left(E - C_1 e^{-\frac{R}{L}t}\right)$$

$$0 = \frac{1}{R}\left(E - C_1 e^{0}\right)$$

$$C_1 = E$$

$$I = \frac{1}{R}\left(E - Ee^{-\frac{R}{L}t}\right)$$

$$I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

c)

$$I = \frac{110}{10}\left(1 - e^{-\frac{10}{20}t}\right)$$

$$I = 11\left(1 - e^{-\frac{1}{2}t}\right)$$

(graph)

d)

i) @ 1 second, 4.33 amps

ii) @ 10 seconds, 10.93 amps

iii) after many seconds, 11 amps

e)

$$I = 0.95(11) = 10.45$$

$$10.45 = 11\left(1 - e^{-\frac{1}{2}t}\right)$$

$$0.95 = 1 - e^{-\frac{1}{2}t}$$

$$-0.05 = -e^{-\frac{1}{2}t}$$

$$0.05 = e^{-\frac{1}{2}t}$$

$$\ln 0.05 = -\frac{1}{2}t$$

$$-2\ln 0.05 = t$$

$$t = 5.9914$$

6 seconds