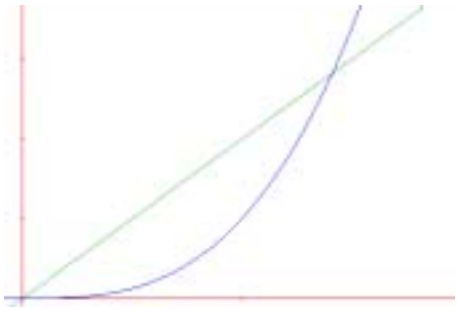
Volumes-by-Cross-Sections Review KEY



The following questions refer to the first-quadrant region bounded by $y = x^3$ and $y = 2x$.

Set-up but do not evaluate the integrals.

Helpful notes for all 10 problems:

$f(x) = x^3$, $f(y) = \sqrt[3]{y}$, $g(x) = 2x$, $g(y) = \frac{1}{2}y$, functions intersect at $(0,0)$ and $\left(\sqrt{2}, 2\sqrt{2}\right)$

All of the rotation problems can be done with washers OR shells. Both answers are given and either is acceptable.

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1. Find the volume of the region generated when rotated about the *x*-axis.

washers: $\pi \int_0^{\sqrt{2}} (2x)^2 - \left(x^3\right)^2 dx$, **shells:** $2\pi \int_0^{2\sqrt{2}} y\left(\sqrt[3]{y} - \frac{1}{2}y\right) dy$

2. Find the volume of the region generated when rotated about the *y*-axis.

washers: $\pi \int_0^{2\sqrt{2}} \left(\sqrt[3]{y}\right)^2 - \left(\frac{1}{2}y\right)^2 dy$, **shells:** $2\pi \int_0^{\sqrt{2}} x\left(2x - x^3\right) dx$

3. Find the volume of the region generated when rotated about $y = -2$.

washers: $\pi \int_0^{\sqrt{2}} (2x+2)^2 - \left(x^3+2\right)^2 dx$, **shells:** $2\pi \int_0^{2\sqrt{2}} (y+2)\left(\sqrt[3]{y} - \frac{1}{2}y\right) dy$

4. Find the volume when cross-sections perpendicular to the *x*-axis are sides of squares.

$$\int_0^{\sqrt{2}} \left(2x - x^3\right)^2 dx$$

5. Find the volume of the region generated when rotated about $x = 2$.

washers: $\pi \int_0^{2\sqrt{2}} \left(2 - \frac{1}{2}y\right)^2 - \left(2 - \sqrt[3]{y}\right)^2 dy$, **shells:** $2\pi \int_0^{\sqrt{2}} (2-x)\left(2x - x^3\right) dx$

6. Find the volume when cross-sections perpendicular to the *y*-axis are legs of isosceles right triangles.

$$\int_0^{2\sqrt{2}} \tfrac{1}{2}\left(\sqrt[3]{y} - \tfrac{1}{2}y\right)^2 dy$$

7. Find the volume of the region generated when rotated about $y = 5$.

washers: $\pi \int_0^{\sqrt{2}} \left(5 - x^3\right)^2 - \left(5 - 2x\right)^2 dx$, **shells:** $2\pi \int_0^{2\sqrt{2}} \left(5 - y\right)\left(\sqrt[3]{y} - \tfrac{1}{2}y\right) dy$

8. Find the volume when cross-sections perpendicular to the *x*-axis are diameters of circles.

$$\int_0^{\sqrt{2}} \pi \left(\frac{2x - x^3}{2}\right)^2 dx$$

9. Find the volume of the region generated when rotated about $x = -3$.

washers: $\pi \int_0^{2\sqrt{2}} \left(\sqrt[3]{y} + 3\right)^2 - \left(\tfrac{1}{2}y + 3\right)^2 dy$, **shells:** $2\pi \int_0^{\sqrt{2}} \left(x + 3\right)\left(2x - x^3\right) dx$

10. Find the volume when cross-sections perpendicular to the *y*-axis are sides of squares.

$$\int_0^{2\sqrt{2}} \left(\sqrt[3]{y} - \tfrac{1}{2}y\right)^2 dy$$